$$p^2=\frac{y^2-c_1^2}{c_1^2}, \text{ or } p=\frac{\sqrt{(y^2-c_1^2)}}{c_1^2}\ldots\ldots(7).$$

Then, substituting for p, and separating,

$$\frac{dy}{\sqrt{y^2-c_1^2}}=\frac{dx}{c_1^2}\ldots\ldots(8). \text{ Integrating, } \log[y+\sqrt{(y^2-c_1^2)}]=\frac{x}{c_1^2}+\log c_2\ \ldots\ldots(9).$$

$$\therefore \frac{y+\sqrt{(y^2-c_1^2)}}{c_2}=e^{x/c_1}\ldots\ldots(10).$$

Transposing and squaring, $y^2-c_1^2=c_2^2e^{2x/c_1}-c_2e^{x/c_1}y+y^2\ldots\ldots(11)$.

$$\therefore y=\frac{c_2^2e^{2x/c_1}+c_1^2}{c_22e^{x/c_1}}\ldots\ldots(12), \text{ or } y=\frac{c_2^2e^{x/c_1}+c_1^2e^{-(x/c_1)}}{2c_2}\ldots\ldots(13).$$

Let $c_2=c_1$, thus moving the origin to the right. Then $y=\dfrac{c_1[e^{x/c_1}+e^{-(x/c_1)}]}{2}$ ....(14), or $y=c_1\cosh(x/c_1)$, which is the equation of the catenary.

Also solved by *J. W. YOUNG*.

87. Proposed by MARY M. BLAINE, B. Sc., Graduate Student, Drury College, Springfield, Mo.

Integrate $(px-y)(py+x)=h^2p$, where $p=dy/dx$.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.

We have $(px-y)(py+x)=h^2p\ldots\ldots(1)$, or

$$p^2xy-py^2+px^2-xy=h^2p\ldots\ldots(2).$$

Multiplying by y and arranging,

$$y^2=\left(y\frac{dy}{dx}\right)^2-\left(\frac{y^3}{x^2}-xy+\frac{h^2y}{x}\right)\frac{dy}{dx}\ldots\ldots(3).$$

Putting $y^2=y'$, $x^2=x'$,

$$y'=x'\left(\frac{dy'}{dx'}\right)^2-y'\frac{dy'}{dx'}+x'\frac{dy'}{dx'}-\frac{dy'}{dx'}h^2\ldots\ldots(4),$$

or $y'=p'x'-\dfrac{h^2p'}{p'+1}\ldots\ldots(5)$, Clairaut's Form, giving $y^2-cx^2=-\dfrac{ch^2}{c+1}\ \ldots\ldots(6)$.

II. Solution by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Divide by p, differentiate and reduce to

$$xy\frac{dp}{dx}+(px-y)p=0.$$

This may be written

$$\frac{\frac{d}{dx}(px-y)}{px-y}=-\frac{1}{y}\frac{dy}{dx},$$

the integral of which is $pxy-y^2=c$.

Eliminating p between this and the given equation,

$$y^2+\frac{c}{c-h^2}=-c.$$

Putting $\frac{c}{c-h^2}=-c'$, $y^2-c'x^2=\frac{c'h^2}{1+c'}$.

This solution may also be obtained by solving $pxy-y^2=c$, obtaining $y^2+c'x^2=c$.

This gives $p=-\frac{c'x}{y}$, which substituted in the given equation makes $c=\frac{-c'h^2}{1+c'}$.

Also solved by the *PROPOSER*.

MECHANICS.

79. **Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.**

The four wheels of a street car are rigidly fixed to their axles so that axles and wheels turn together. Is it more advantageous to apply the brakes to the front or to the rear wheels, supposing the brakes to block the wheels in each case?

Solution by the PROPOSER.

The question may be answered by treating the problem as a statical one, thus: Suppose the car placed upon an inclined plane and let us inquire in which case may the plane be raised to the greater angle before slipping begins. Let $2a$ be the distance between the centers of the wheels, each of radius c, b the distance of the center of gravity, G of the car above (or below) this line of centers, w the weight of each of the trucks, w_1 the weight of car.

Take the case first when brakes are applied to rear wheels, there being in this case, of course, no friction between the front wheels and plane. Consider the figure as consisting of two rigid bodies, viz, the front trucks, and the car with the rear trucks. The forces acting upon the front trucks are their weight w, the reaction R of the plane, and a force, P, at O obliquely downward, which is the resultant of w and the backward pull of the car and rear wheels. Upon the car at this point O there will also be an equal and opposite force to P, the resultant of R and w.